

AVERAGE LIMITS

Scenario

Think of two numbers, say 12 and 6, and make a sequence where the next number is always the average of the previous two numbers.

The average of 12 and 6 is $(12 + 6) \div 2 = 9$

The sequence is now 12, 6, 9, ...the next number is the average of 6 and 9 $(6 + 9) \div 2 = 7.5$

The sequence is now 12, 6, 9, 7.5, ...

- Continue this sequence for 10 terms.
- Describe any patterns you notice. You should notice that the sequence gets closer and closer to 8.
- Continue the sequence for 5 more terms and see what happens.

We say the “limit” of the sequence is 8.

If the sequence does not get closer and closer to 8 you have made a mistake, go back and check your calculations.

- By finding at least the first 10 terms in each sequence, find the “limit” of the sequences that start with the following numbers.

a) 6, 12, ... b) 9, 3, ... c) 3, 9, ... d) 6, 3, ... e) 3, 6, ...

This process takes a long time with a calculator, so try to use an alternative method using Excel or a programmable calculator to generate the results more quickly.

- Show any Excel formulas or calculator programs that you use.
- Tabulate your results for the first two numbers. We shall call them a and b and the Limit L.

Sequence	a	b	L
1	12	6	8
2	6	12	...
3	9	3	...
....

- Can you find a rule connecting the first two numbers, a and b, and the “limit” L?
- If L is a linear combination of a and b, set up two simultaneous equations and find the rule.
- Check your rule by using other starting values. (Try values that are not multiples of 3!)

Example 3

- Can you prove your rule algebraically?

E.g. If a sequence starts with a , b , the next term is the average of a and b , which is:

$$\frac{1}{2} (a + b) = \frac{1}{2}a + \frac{1}{2}b$$

The average of b and $\frac{1}{2}a + \frac{1}{2}b$ is $\frac{1}{2} (b + \frac{1}{2}a + \frac{1}{2}b) = \frac{3}{4}b + \frac{1}{4}a \dots$ (work out 10 terms)

- Can you justify your findings?
- Reflect on the different methods used in this project.

Extending The Problem

- Can you explore what happens with a sequence where the next value is the average of the first 3 starting numbers? Start with 18, 12, 6, ...
- Can you explore what happens with a sequence where the next value is the average of the first 4 starting numbers? Start with 40, 30, 20, 10 ...